Transformation of seismic discontinuous waves by hyperboloid interfaces in anisotropic elastic media

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Abstract. In this paper, interaction of discontinuous waves with hyperboloid heterogeneities in anisotropic elastic media is investigated. It is shown that the interactions are accompanied by formation of reflected and refracted quasi-longitudinal and quasi-shear discontinuous waves which can be focused or scattered by elastic "mirrors" and "lenses" depending on their geometric outlines and acoustical properties. The set up problem solutions can be used for discovering the most and least seismically hazardous zones in the earth crust and for interpretation of geophysical data about geological rock structures.

Key words: Anisotropic media; Discontinuous waves; Elastic mirrors and lenses; Scattering.

1. Introduction

Elastic waves induced by seismic phenomena, explosions, and other causes are studied in seismology for establishment of general regularities of earthquakes and processes associated with them, as well as for identification of the Earth's crust structure. When investigating the seismic effects, the important role is assigned to dynamic theoretical models including discontinuous waves. These waves can be provoked by volcanic phenomena or by collapses and ruptures of the pre-stressed abyssal rock layers (Eiby 1980, Kasahara 1981). As a rule, they have a clearly defined shock character caused by the fact that the rock medium is in a non-deformed state ahead the moving surface of the wave front, the functions of stresses and strains have finite values behind the surface, and they experience discontinuities at the surface itself. Different practical questions of the seismic wave interacting with rock inhomogeneities are studied by Arif et al (2012), Bidgoli and Jing (2014), Brule (2014), Dolan and Haravith (2014).

The ability of the tectonic medium to permit the passage of the discontinuous waves is connected with the orderliness of its rock structures. The more ordered is the structure the more pronounced shock type wave with the less thickness of the transition layer at its front can be allowed to pass through the rock medium. At the same time, the discontinuous character of the propagating wave is associated also with the scaling factor of the dynamic process which is determined by the relationships between dimensions of the elastic medium boundary surfaces, sizes of the medium particles and values of geometrical parameters of the moving wave front surface.

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For this reason, every so often the phenomena of seismic wave propagation in the tectonic structures can be simulated with the use of discontinuous functions.

In the event that during the wave front propagation its smoothness is broken owing to its surface rearranging, the peaks of strains and stresses are initiated at the sites of the geometric singularities formed. The effects of this sort are most typical for elastic materials possessing properties of anisotropy and heterogeneity inherent in rock media. In this case, the additional distortions of the wave front regularity come into being at the tectonic anomalies in the shapes of lenses and distortions of the rock layers. These anomalies can concentrate or scatter the wave energy depending on their outlines and ratios between values of the physical parameters of the media.

The discussed phenomena are usually characterized by short duration of highly intensive initial field of pressure, which at the original stage of time is concentrated, as a rule, in a small domain adjacent to the zone of impact initiation of the wave and by transformation of the wave front surface as it propagates. Inasmuch as in this case the boundary of the domain chosen for calculation evolves with the wave front progress, the solution has to be found in the family of discontinuous functions evolving in time. So the traditional classical and numerical methods turn out to be of low efficiency for analysis of the similar processes.

In solution of these type problems a prominent role is played by the methods of geometrical optics (Fedorov 1968, Karal and Keller 1959, Kravtsov and Orlov 1980, Ogilvy 1990) in non-dispersive media. They are correlated with application of a ray coordinate system wherein families of the coordinate surfaces coincide with the evolving surfaces of the non-stationary waves. Formally, this approach is realized through representation of the wave equation solution in terms of a ray series. With its use, an eikonal equation and transport equations system are constructed. The former is a non-linear partial differential equation describing the front surface and the ray aggregate which is referred to as "kinematic equation". The transport equations constitute a system of linear partial equations. They determine the field functions at the front surface and behind it which are referred to as "dynamic equations". With this approach being in use, the special cases can be distinguished, when the aggregates of the rays are produced, which have common envelope (caustics), where the rays are focused and the field intensity increases indefinitely. In geometrical optics, the caustics classification is performed on the basis of the theory of singularities of differentiable mappings - the theory of catastrophes (Arnold 1990, Arnold et al. 1984, Poston and Stewart 1978).

In studies of discontinuous (shock type) waves propagation in elastic media, the greatest attention is, as a rule, devoted to geometrical construction of the evolving field-function discontinuity surfaces and calculation of the discontinuity magnitudes, which provide the most complete information about the wave front transformation and the intensity of an impulse carried by the wave at each point of the front surface. Because of this, usually the greatest attention is placed on the zeroth approximation of the ray method [19], providing good quantitative description of the wave phenomenon in a small vicinity of the wave front. Its application allows one to construct the evolving front, to determine the wave polarization vector at every point of its surface, to calculate the discontinuities magnitudes of the functions of strains and stresses and also the wave phase as functions of the ray coordinates. With this method, the zeroth term of the ray series is taken into account, which can be calculated independently of other ones.

The zeroth approximation of the ray method used jointly with the locally plane approach [18, 19] allows one to state the problem about interaction of the shock waves with interfaces between elastic media possessing different mechanical properties. This problem is associated with the necessity to construct kinematically the front surfaces of reflected and refracted waves with different polarizations and to calculate dynamical parameters of the field discontinuities on these fronts.

The important advantage of this approach is associated with the fact that in order to find the zeroth term of the expansion it is not necessary to solve any differential equation, because the value of the stress function discontinuity at the wave front is expressed through its initial value and a ratio between geometrical divergences of the rays at the initial and terminal points of the ray. In this event, it is not always essential to calculate the exact values of the discontinuity, but is possible to restrict ourselves to evaluating whether their values enlarge or diminish as the front point moves along the ray. This evaluation can be done with the help of visual analysis of the system of rays and fronts through separation of the ray focusing points and zones of their divergence. General regularities of the ray transformations after their interactions with interfaces between homogeneous isotropic media are determined by focal properties of their surfaces and acoustic stiffnesses of the elastic media. Therefore, it is important to consider the reflectors of simplest shapes and to analyze wave diffractions in them. Amidst such surfaces, there are rotary ellipsoids, Fig.1a, hyperboloids, Fig.1b, and paraboloids, Fig.1c.

Axial cross-section of the ellipsoid is ellipse with foci F_1 and F_2, not lying in the rotary axis Ox_2. For this reason, it is not possible to prognosticate peculiarities of the ray transformations by ellipsoidal heterogeneities. Gulyayev and Ivanchenko (2003, 2004) and Gulyayev et al (2004) studied the problem of the wave front reshaping by free ellipsoidal surfaces, interfaces, concave and convex lenses. It was revealed that the effects of the wave focusing and scattering can be attained if the anisotropy parameters of the elastic media are duly chosen.

Focus F of a hyperboloid surface is locate in its rotary axis, Fig.1b, and so it can be anticipated that the property of well-ordered transforming of the waves by this interface is exhibited more distinctly. This hypothesis is not tested till now and is of some interest.

The focusing properties of the paraboloidal surface, Fig.1c, in homogeneous isotropic media are well known but influence of the medium anisotropy on this effect is not also analyzed. In this paper, the problem associated with analysis of focusing and scattering plane discontinuous waves by hyperboloidal interfaces and lenses is examined. The effects of wave transforming by paraboloidal surfaces will be analyzed later.

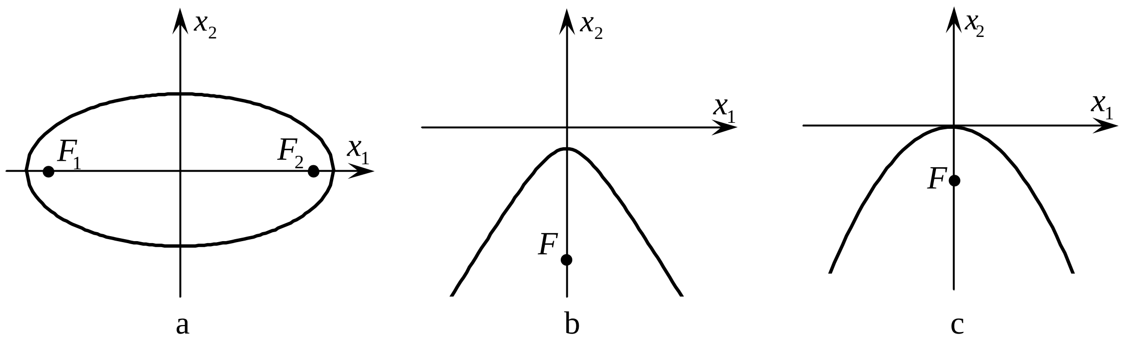


Figure 1 Types of hypothetic interfaces in rock media: ellipsoidal (a); hyperboloidal (b); paraboloidal (c).

In seismology, these questions are topical for investigation of wave processes occurring in the earth's crust and for description of behavior of seismic waves in the vicinity of tectonic inhomogeneities, where the waves can endure the effects of focusing or scattering. These effects manifest themselves most clearly in the convex and concave parts of the interfaces between rock structures.

It is known that there are no ways of prognosticating and eliminating the earth-quakes, but using the stated problem solutions it is possible to find the tectonic regions where the natural seismic waves can focus and concentrate their energy provoking collapse of above-ground and underground constructions or where the waves disperse without damage for the environment.

In the course of seismic reconnaissance of mineral resources, the considered results are useful for theoretical interpretation of geophysical data about the explored geological rock structures. The problem solution can also be used for analysis of explosion wave influence on environment and for elaboration of rational methods for pursuance of explosion works eliminating the possibility of artificial earthquakes generation at predetermined regions.

2. Technique of numerical simulation

In the analysis of discontinuous waves propagation in anisotropic elastic media, the equations of dynamic equilibrium of its particles in the Cartesian coordinate system x_1, x_2, x_3 are chosen in the form:

$$\sum_{k,p,q=1}^{3} \frac{c_{ik,pq}}{\rho} \frac{\partial^2 u_q}{\partial x_k \partial x_p} - \frac{\partial^2 u_i}{\partial t^2} = 0 \qquad (i = 1,2,3) \qquad (1)$$

where ρ is the density of the medium; u_1, u_2, and u_3 are elastic displacements; t is time; and $c_{ik,pq}$ are the components of the tensor of elastic parameters of the medium. Generally the $c_{ik,pq}$ tensor has 81componens. In the case under study, the so called “transversely isotropic media” is characterized by 5 elastic parameters. Due to the symmetry of this tensor with respect to the Ox_2 axis and the fact that it has only five irreducible parameters, it can be brought to a two dimensional form:

$$\left\| C_{\alpha\beta} \right\| = \left\| \begin{matrix} C_1 & 0 \\ 0 & C_2 \end{matrix} \right\|,$$

where (2)

$$C_1 = \left\| \begin{matrix} \lambda+2\mu & \lambda-l & \lambda \\ \lambda-l & \lambda+2\mu-p & \lambda-l \\ \lambda & \lambda-l & \lambda+2\mu \end{matrix} \right\|, \quad C_2 = \left\| \begin{matrix} \mu-m & 0 & 0 \\ 0 & \mu & 0 \\ 0 & 0 & \mu \end{matrix} \right\|,$$

where λ and μ are the Lame's coefficients and l, m and p are the anisotropy parameters, which distinguish the medium under consideration from the isotropic one with $l = m = p = 0$.

The solution to system of Eqs. (1) is represented in the form of a plane monochromatic wave

$$\mathbf{u} = \mathbf{A} \cdot e^{ik(\mathbf{n}\cdot\mathbf{r} - \mathrm{v}t)} \tag{3}$$

with a phase number k and a phase velocity $\mathbf{v}$. The phase fronts of this wave will be surfaces of constant phase $\mathbf{n}\cdot\mathbf{r} - \mathrm{v}t = const$ locally perpendicular to the unit vector $\mathbf{n}$ and traveling with velocity $\mathbf{v} = \mathrm{v}\cdot\mathbf{n}$. The magnitude of the phase velocity $\mathbf{v}$ and the vector of wave polarization $\mathbf{A}$ are determined from the homogeneous system of algebraic equations [7, 19]

$$\sum_{k,p,q=1}^{3} \frac{c_{ik,pq}}{\rho} n_k n_p A_q - \mathrm{v}^2 A_i = 0 \qquad (i = 1,2,3) \tag{4}$$

as eigenvalues and eigenvectors of the symmetric and positively defined Christoffel matrix $\Lambda_{iq} = \sum_{k,p=1}^{3} c_{ik,pq} \rho^{-1} n_k n_p$ where $i, q = 1,2,3$. The condition of existence of a nonzero solution for the homogeneous system in Eqs. (4) is written in the form of a cubic equation in the squared phase velocity as

$$\left| \sum_{k,p=1}^{3} \frac{c_{ik,pq}}{\rho} n_k n_p - \mathrm{v}^2 \delta_{iq} \right| = 0. \tag{5}$$

For any preliminarily chosen direction of the unit normal $\mathbf{n}$ to the front, the roots of this equation are three positive numbers that allow the determination of the phase velocity magnitudes and their arrangement in descending order: $\mathrm{v}_1(\mathbf{n}) > \mathrm{v}_2(\mathbf{n}) \geq \mathrm{v}_3(\mathbf{n}) > 0$. The maximum propagation velocity corresponds to the quasi-longitudinal wave, while the remaining velocities correspond to two quasi-transverse waves with different polarizations. The polarization vector $\mathbf{A}^{(r)}$ for each of the three $(r = 1,2,3)$ waves possible in anisotropic media can be found by substituting the corresponding squared phase velocity into system of Eqs. (4) and determining the eigenvector of the matrix of coefficients of the system of equations

$$\sum_{k,p,q=1}^{3} \frac{c_{ik,pq}}{\rho} n_k n_p A_q^{(r)} - [\mathrm{v_r}]^2 A_i^{(r)} = 0 \qquad (i = 1,2,3). \tag{6}$$

The time-dependent phase front surface of an elastic wave is described by the relationship $\tau(x_1, x_2, x_3) - t = 0$ where τ is some function satisfying the first-order partial differential equation (Petrashen 1980)

$$\sum_{i,k,p,q=1}^{3} \frac{c_{ik,pq}}{\rho} \frac{\partial \tau}{\partial x_k} \frac{\partial \tau}{\partial x_p} A_q^{(r)} A_i^{(r)} = 1, \tag{7}$$

which extends the eikonal equation of geometrical optics to the case of elastic waves in anisotropic media.
Partial derivatives of the function τ with respect to the Cartesian coordinates are the components of the refraction vector $\mathbf{p}$ and are determined by the formulas

$$p_k \equiv \partial \tau / \partial x_k = \mathbf{n} / \mathrm{v}_r(\mathbf{n}) \qquad (k = 1,2,3)$$

To construct the frontal surfaces of an elastic nonstationary wave in a homogeneous anisotropic medium $(\rho = const, c_{ik,pq} = const)$, we must find solutions to Eqs. (7) that can be reduced to a system of ordinary differential equations by using the method of characteristics:

$$\xi_k = \frac{dx_k}{d\tau} = \frac{1}{\rho} \cdot \sum_{i,p,q=1}^{3} c_{ik,pq} p_p A_q^{(r)} A_i^{(r)}, \qquad \frac{dp_k}{d\tau} = 0 \qquad (k = 1,2,3) \tag{8}$$

In the system of Eqs. (8), three first equations allow the determination of three components of the ray velocity vector $\boldsymbol{\xi} = \boldsymbol{\xi}^{(r)}(\mathbf{n}, x_k)$ along which the wave travels. Another group of equations shows that these rays are rectilinear in a homogeneous anisotropic medium.

Hence, the kinematic problem of constructing the evolving front of a non-stationary shock wave in a homogeneous anisotropic medium is reduced to the construction of a system of rectilinear rays whose directions satisfy Eqs. (8) and correspond to a given sequence of normal vectors $\mathbf{n}$ to the wave front. For a certain value of $t = const$, the locus of the points lying on these rays and located at a distance $\xi(\mathbf{n}) \cdot t$ from the elastic wave source, forming the front surface.

The built up system of rays and fronts allows to proceeding to the determination of the wave intensity in the vicinity of its front. For the realization to be performed, it is convenient to use Eqs. (1) solution-expansion in series along a ray as follows:

$$u_q = \sum_{m=0}^{\infty} u_q^{(m)}(x_1, x_2, x_3) f_m[t - \tau(x_1, x_2, x_3)] \qquad (q = 1,2,3) \tag{9}$$

where the functions f_m, satisfying the correlations $f'_m(y) = f_{m-1}(y)$, are supposed to be multiplied by the Heaviside function and to possess discontinuities of their derivatives (23Petrashen 1980).

If the problem of investigation of the wave behavior in the front nearest neighborhood is set up, only one term

$m=0$ is retained in Eqs. (9) and for the vector $\mathbf{u}^{(0)}$ to be calculated, the system of homogeneous equations

$$\sum_{k,p,q}^{3}\lambda_{ik,pq}p_k p_p u_q^{(0)} - u_i^{(0)} = 0 \qquad (i=1,2,3) \tag{10}$$

is used. Its solution is represented in the form [19]:

$$u_q^{(0)} = \frac{c_0(\alpha,\beta)\cdot A_q^{(r)}(\alpha,\beta,\tau)}{\sqrt{J(\alpha,\beta,\tau)}} \qquad (q=1,2,3) \tag{11}$$

where α, β, τ is the system of ray coordinates and the functional determinant $J=\partial(x_1,x_2,x_3)/\partial(\alpha,\beta,\tau)$ of the transformation of the ray coordinates into Cartesian ones is the measure of the ray divergence in the ray tube.

The presented correlations permit to trance the evolution of a discontinuous wave front and to calculate magnitudes of the field functions discontinuities on its surface outside the interface between anisotropic elastic media with differing properties. The interaction of rays and wave fronts with the boundaries between anisotropic media with different physical parameters makes their geometries much more complicated. In the general case, the front incident on the boundary generates three refracted and three reflected fronts of differently polarized elastic waves.

Consider a plane discontinuous wave traveling along the Ox_2 axis and passing through a convex anisotropic lens, and assume that the Ox_2 axis coincides with the symmetry axes of both transversely isotropic medium and lens. This is a special direction in which the vectors of the ray and phase velocities coincide, so that the wave appears purely longitudinal. Because of the axial symmetry of the problem, an investigation of the behavior of traces of front surfaces in some plane containing the Ox_2 axis appears to be sufficient for the solution. We distinguish a free surface of elastic anisotropic medium, Fig.2a, and interface, Fig.2b, between two transversely isotropic media with different elastic parameters: the initial medium (with the incident wave), whose parameters are marked with subscript *I*, and the internal medium of the lens marked with subscript *II*, Fig.2b. At every point M of the first boundary surface of the lens (boundary G_1 between elastic media *I* and *II)* the incident ray produces a beam of two refracted and two reflected rays whose directions and phase velocities satisfy Snell's law [7]

$$\frac{\sin(\gamma)}{\mathrm{v}} = \frac{\sin(\Theta_\nu - \gamma)}{\mathrm{v}_\nu(\Theta_\nu)} = \frac{\sin(\overline{\Theta}_\mu + \gamma)}{\overline{\mathrm{v}}_\mu(\overline{\Theta}_\mu)} \qquad (\nu,\, \mu = 1,2,3) \tag{12}$$

where γ is the angle of inclination of the tangent to the surface G_1 at the point M of ray incidence; Θ_1 and Θ_2 are the angles between the Ox_2 axis and the directions of the phase velocity vectors of the quasi-longitudinal qP and quasi-transverse qS waves reflected into medium *I*; Θ_1 and Θ_2 are the corresponding angles for waves refracted in medium *II* (in the lens); and v, v_ν, and $\bar{\mathrm{v}}_\mu$ are the phase velocities of the incident longitudinal wave and reflected and refracted waves (subscripts 1 and 2 correspond to quasi-longitudinal and quasi-transverse waves, respectively). The characteristic feature of Snell's law of Eqs. (12) for anisotropic media is that the denominators v_ν, and $\bar{\mathrm{v}}_\mu$ are explicit functions of the corresponding angles Θ_ν and $\overline{\Theta}_\mu$ and implicit functions of the angle γ as are the numerators.

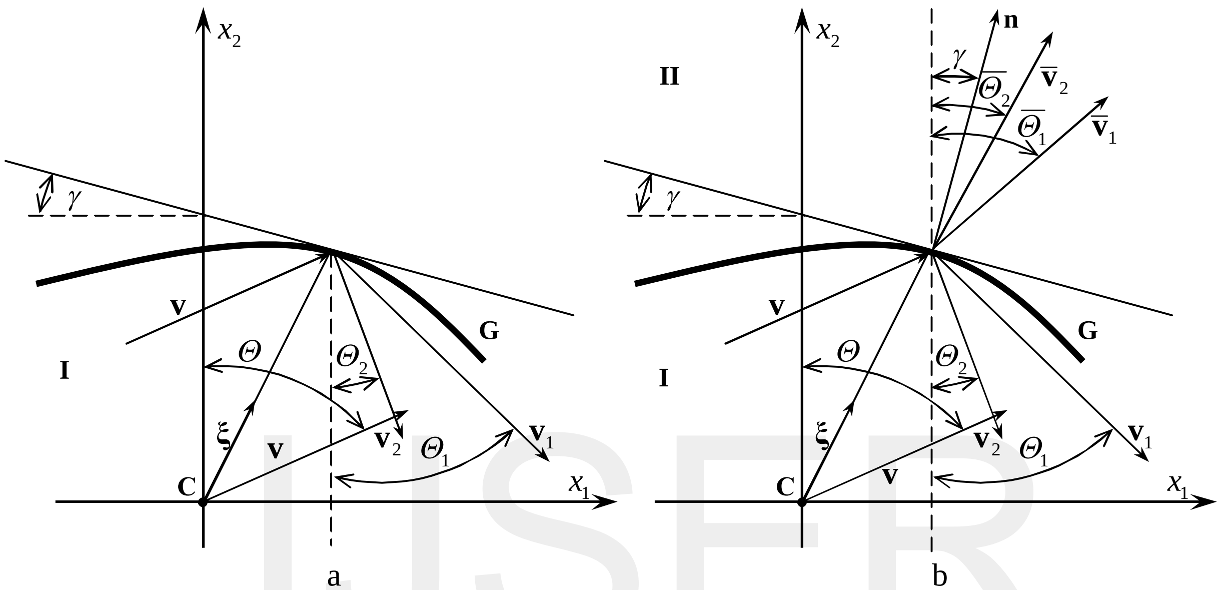


Figure 2 Schematic diagrams of orientations of the phase velocity vectors at the free surface of elastic medium (a) and interface G between elastic media (b).

The refraction and reflection angles Θ_ν and $\overline{\Theta}_\mu$ $(\nu, \mu = 1,2,3)$ at a point M of boundary G_1 are obtained from the nonlinear system of Eqs. (12), which is solved using the Newton method combined with the algorithm of solution continuation with respect to a parameter [17]. The angle of inclination of the tangent γ appears to be a convenient choice for the leading parameter. With such a choice, for the first equation of system of Eqs.(12) with certain known parameter $\gamma = \gamma^n$ and vectors $\mathbf{v}_\nu^n$, a small increment of the leading parameter $\Delta\gamma^n$ will cause the following increments of the pointing angles of elastic waves reflected into medium *I*:

$$\Delta\Theta_\nu^n = \frac{\cos\gamma \cdot \mathrm{v}_\nu\left(\Theta_\nu^n\right) + \cos\left(\Theta_\nu^n - \gamma\right)\cdot \mathrm{v}}{\cos\left(\Theta_\nu^n - \gamma\right)\cdot \mathrm{v} - \sin\gamma \cdot \partial \mathrm{v}_\nu\left(\Theta_\nu^n\right)/\partial\Theta_\nu} \cdot \Delta\gamma + r_\nu \qquad (13)$$

where $r_\nu = \sin\left(\Theta_\nu^n - \gamma\right)\cdot \mathrm{v} - \sin\gamma \cdot \mathrm{v}_\nu\left(\Theta_\nu^n\right)$ are the residuals of the equations at the considered step.

Rays of quasi-longitudinal and quasi-transverse waves outgoing from point M are incident on the boundary surface G_2. At the points M_p and M_s of incidence on surface G_2 each wave produces new beams of quasi-longitudinal and quasi-transverse waves refracted into medium *I* and reflected into the lens (medium *II*). Phase velocities and vector directions of the rays of every beam again satisfy Snell's equations, which now have the form:

$$\frac{\sin(\Theta-\varphi)}{\mathrm{v}(\Theta)}=\frac{\sin(\Theta_\nu+\varphi)}{\mathrm{v}_\nu(\Theta_\nu)}=\frac{\sin(\overline{\Theta}_\mu-\varphi)}{\overline{\mathrm{v}}_\mu(\overline{\Theta}_\mu)} \qquad (\nu,\mu=1,2) \tag{14}$$

The solution of system of Eqs. (14) is also performed according to the step-by-step procedure. By way of example, a small variation $\Delta\varphi^n$ of the angle of inclination of the tangent to surface G_2 will cause increments of the pointing angles of phase velocity vectors of the refracted waves of both types:

$$\Delta\Theta_\nu^n=\frac{f_1\Delta\Theta+f_2\Delta\varphi}{f_3}+r_\nu^n \tag{15}$$

Here, we introduced the functions

$$f_1=\sin(\Theta_\nu^n+\varphi)\cdot\frac{d\mathrm{v}(\Theta)}{d\Theta}-\cos(\Theta-\varphi)\cdot\mathrm{v}_\nu^{\mathrm{n}},$$
$$f_2=\cos(\Theta-\varphi)\cdot\mathrm{v}_\nu^{\mathrm{n}}(\Theta_\nu^n)+\cos(\Theta_\nu^n+\varphi)\cdot\mathrm{v},$$
$$f_3=\sin(\Theta-\varphi)\cdot\frac{d\mathrm{v}_\nu^{\mathrm{n}}(\Theta_\nu^{\mathrm{n}})}{d\Theta_\nu^{\mathrm{n}}}-\cos(\Theta_\nu^n+\varphi)\cdot\mathrm{v}(\Theta)$$

and the residual of Eqs. (11) is $r_\nu^n=\sin(\Theta_\nu^n+\varphi)\cdot\mathrm{v}-\sin(\Theta-\varphi)\cdot\mathrm{v}_\nu^{\mathrm{n}}(\Theta_\nu^{\mathrm{n}})$. The implementation of successive calculations by formulas like of Eqs. (13) and (15) requires the knowledge of some initial state γ, v, Θ_ν^0 and $\mathrm{v}_\nu(\Theta_\nu^0)$. In the case of the axially symmetric lens under consideration, a convenient choice of the initial direction is $\gamma=0$, which corresponds to constructing a family of reflected and refracted rays beginning with the ray directed along the Ox_2 axis, because this ray produces rays directed along this very axis at both lens surfaces. For nonzero denominators, formulas like of Eqs. (13) and (15) allow the determination of a unique set of increments for pointing angles of all phase velocity vectors of both types of waves at both lens surfaces. Angles γ or which the denominator on the right-hand side of Eqs. (13) and (15) vanishes,

$$\begin{gathered}\cos(\Theta_\nu^n-\gamma)\cdot\mathrm{v}-\sin(\gamma)\frac{\partial\mathrm{v}_\nu(\Theta_\nu^n)}{\partial\gamma}=0,\\ \sin(\Theta-\varphi)\cdot\frac{d\mathrm{v}_\nu^{\mathrm{n}}(\Theta_\nu^{\mathrm{n}})}{d\Theta_\nu^{\mathrm{n}}}-\cos(\Theta_\nu^{\mathrm{n}}+\varphi)\cdot\mathrm{v}(\Theta)=0 \qquad (\nu=1,2)\end{gathered} \tag{16}$$

are the conditions of bifurcation of the solution. The solution continuation through this state requires that the terms of the second (third, and so on as required) order should be added to these equations.

Condition of Eqs. (16) of a possible non-uniqueness of the solutions to system of Eqs. (13), (15) corresponds to the convergence (contact) and intersection of reflected and refracted rays at one lens surface. These effects can be accompanied by the phenomenon of quasi-total internal reflection [9].

Moving away from lens surfaces, rays can touch and cross one another forming the envelopes of ray families called caustics. Since the singularities of the wave front appear in caustics, the focusing occurs at them, which is accompanied by an infinite growth of the stress field intensity at the points of geometric singularities (in the framework of the theory of ideal elasticity).

The variation of the strain discontinuity at the moving front behaves depending on the geometry of the front surface and is characterized by the geometric divergence of rays, which is a function of $\mathbf{n}$ for anisotropic media. The geometric divergence of rays can be determined by the formula [19].

$$L = L(\alpha,\beta,\tau) = c(\alpha,\beta)\sqrt{\frac{|J(\alpha,\beta,\tau)|}{\xi(\alpha,\beta,\tau)}} \qquad (17)$$

where $c(\alpha,\beta)$ is a constant coefficient and $J(\alpha,\beta,\tau)$ is the Jacobian of the transformation of the ray coordinate system to the Cartesian coordinate system. It is used in formula of Eqs. (11). The Jacobian is calculated by the formula

$$J(\alpha,\beta,\tau) = (\mathbf{z_1}[\mathbf{z_2}\times\mathbf{z_3}]) = \begin{vmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{vmatrix} \qquad (18)$$

where $\mathbf{z_1} = \sum_{k=1}^{3}\frac{\partial x_k}{\partial\alpha}\mathbf{i}_k$, $\mathbf{z}_2 = \sum_{k=1}^{3}\frac{\partial x_k}{\partial\beta}\mathbf{i}_k$ and $\mathbf{z}_3 = \sum_{k=1}^{3}\frac{\partial x_k}{\partial\tau}\mathbf{i}_k$ are the coordinate vectors of the curvilinear ray coordinate system α,β, and τ can be determined numerically by using the finite difference scheme for calculating partial derivatives.

3. Results and their discussions

The outlined technique of ray method and conception of the locally plane approach were used for computer simulation of the phenomena of reflection and refraction of discontinuous qP-waves in the course of their interaction with hyperboloid heterogeneities in transversely isotropic elastic media. The physical parameters of the media are chosen to be equal $\lambda_1 = 4.97\cdot10^{10}\,Pa$, $\mu_1 = 3.91\cdot10^{10}\,Pa$, $l_1 = -0.4\cdot\lambda_1$, $m_1 = 0.2\cdot\mu_1$, $p_1 = 0.1\cdot(\lambda_1 + 2\mu_1)$, $\rho_1 = 2650\,kg/m^3$, $\lambda_2 = 3.41\cdot10^9\,Pa$, $\mu_2 = 1.36\cdot10^{10}\,Pa$, $l_2 = -0.4\cdot\lambda_2$, $m_2 = 0.2\cdot\mu_2$, $p_2 = 0.1\cdot(\lambda_2 + 2\mu_2)$, $\rho_2 = 2760\,kg/m^3$. At given values of the media characteristics, their physical properties are symmetrical relative to the Ox_2 axis, so purely longitudinal P-waves and purely shear S-wave are free to move along it.

Firstly consider the case when plane discontinuous P-wave 1, propagating along the Ox_2 axis in medium *I*, interacts with free boundary hyperboloid surfaces, Fig.3. Their generatrices are determined by the equation

$$x_2 = \pm \frac{b}{a}\sqrt{a^2 + (x_1)^2} + h\,, \qquad (19)$$

where $b = 150\,m$ is the semi-distance between the apices of the hyperbolas of Eqs. (19); the ratio $b/a = 0.5$ determines the angle $\alpha = 2arctg(a/b)$ between the hyperbola asymptotes; h is the hyperbola displacement along the Ox_2 axis, it assumes value 12m for the case in Fig.3a and value 9m for Fig.3b; signs " - " and " + " correspond to the convex and concave surfaces, Fig.3.

As a consequence of the incident wave diffraction at the free surface G, quasi-longitudinal qP-wave 2 and quasi-shear qS-wave 3 are generated. In the first case, Fig.3a, the reflected rays of both types are focused. As this takes place, the focal length of qP-wave 2 turned out to be far less than is the one of the qS-wave. Besides, its focus is not clearly defined and it falls outside the limits of Fig.3a.

According to correlations of Eqs. (9-11), (17), (18), the points where the rays intersect are connected with geometric singularities [12, 13]. At these points the wave intensity tends to infinity. The focusing effect can be also peculiar to refracted waves if the hyperboloid surface G interfaces media *I* and *II,* Fig.4. In this case, if the G surface is convex, Fig.4a, the refracted qP-wave 2 scatters while the appropriate qS-wave 3 focuses.

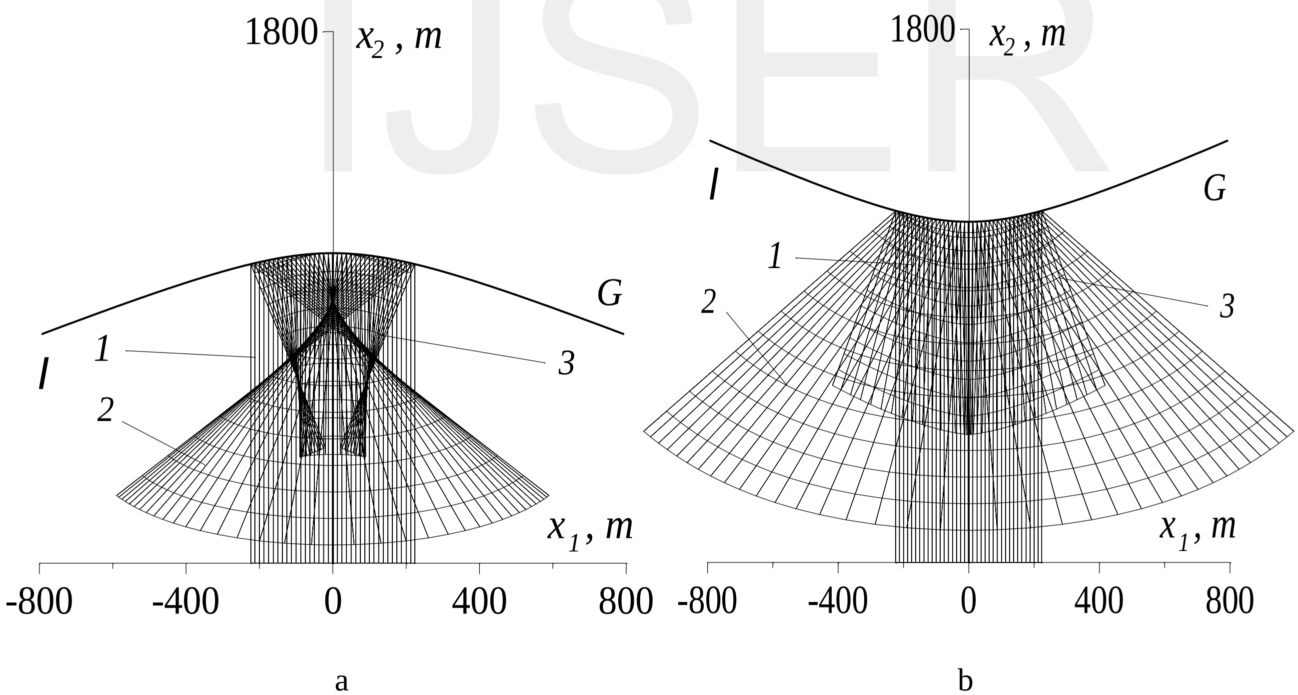


Figure 3 Focusing and scattering plane discontinuous waves by hyperboloid free surface.

At the same time, both of the reflected waves 4,5 are focused, though focal zone of qS-wave 5 is located in a more remote range. For the concave interface G refracted qP-wave 2 focuses and qS-wave 3 scatters, whereas both of reflected waves 4 and 5 scatter, Fig.4b.

The algorithm suggested above was used to investigate the diffraction of a plane longitudinal wave by a convex axially symmetric lens bounded by hyperboloidal surfaces whose traces on the symmetry plane Ox_1x_2 are

$$G_1: \quad x_2 = 200 + 250\sqrt{1+\left(x_1/900\right)^2} \, ,$$

$$G_2: \quad x_2 = 1200 - 150\sqrt{1+\left(x_1/300\right)^2}$$

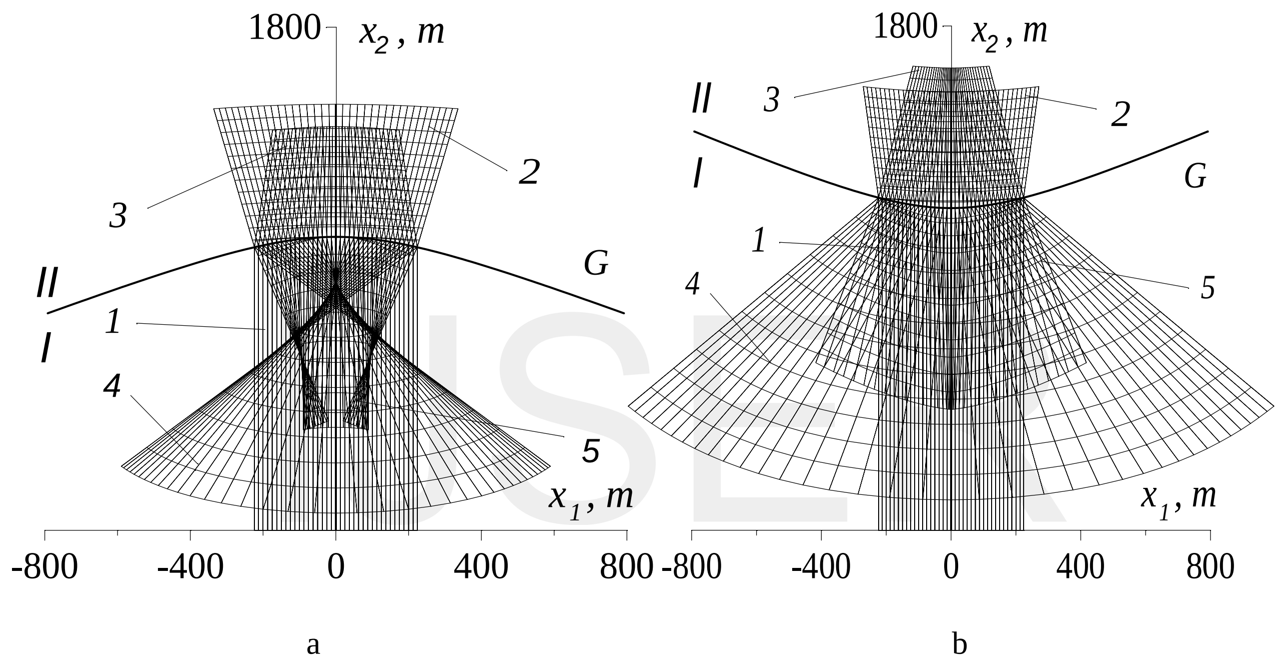


Figure 4 Focusing and scattering plane discontinuous waves by hyperboloid interfaces.

Firstly, the case was considered when the ambient medium and the lens have the properties of materials *I* and *II*, respectively. Fig.5a shows the system of rays belonging to the longitudinal incident wave 1, rays of quasi-longitudinal wave 2 refracted into the lens and rays of quasi-longitudinal wave 3 produced by the latter in medium *I* behind the lens. In Fig.5a, we did not show quasi-longitudinal rays reflected from both lens surfaces, all quasi-transverse rays and evolutions of fronts (excluding the fronts of the quasi-longitudinal wave behind the lens). It is seen that notwithstanding the fact that the lens surfaces have differing curvatures, the initially parallel rays of the elastic wave are focused after their passage through the acoustically softer lens in the same way as light rays in geometrical optics. But the lens focus is not a point; instead, each pair of symmetric rays has its own point of intersection at the symmetry axis, and these intersection points form a focusing zone. When the places of elastic media *I* and *II* are interchanged with the parameters used above, the lens under consideration becomes divergent, Fig.5b. If a lens is biconcave its abilities to focus or scatter wave rays change to apposite ones in comparison with biconvex lenses. Fig.6 corresponds to the concave lens with parameters of surface $G_1 : a_1 = 600m,\ b_1 = -150m$ and $h_1 = 600m$ and $G_2 : a_2 = 600m,\ b_2 = 150m$ and $h_2 = 400m$.

In tectonic structures, the curved layers of their rocks can be one of their typical anomalies. In this connection it is important to analyze how the layer curvature influences on the character of wave front transformation when it passes through the layer. In Fig.7a, the curvilinear layer of rock medium *II* is bounded by surfaces G_1 and G_2 with respective parameters $a_1 = 300m,\ b_1 = 150m$ and $h_1 = 400m$, $a_2 = 900m,\ b_2 = 250m$ and $h_2 = 800m$. In Fig.7b, the layer is turned over and it has properties of medium *I*. It can be seen, that in both cases the layers focus the longitudinal waves.

It is pertinent to note that according to Eq. (18) the $J(\alpha, \beta, \tau)$ values tend to zero at focal zones, so as Eq. (11) testifies, the stress discontinuities acquire infinite values in their vicinities (in the framework of the theory of ideal elasticity).

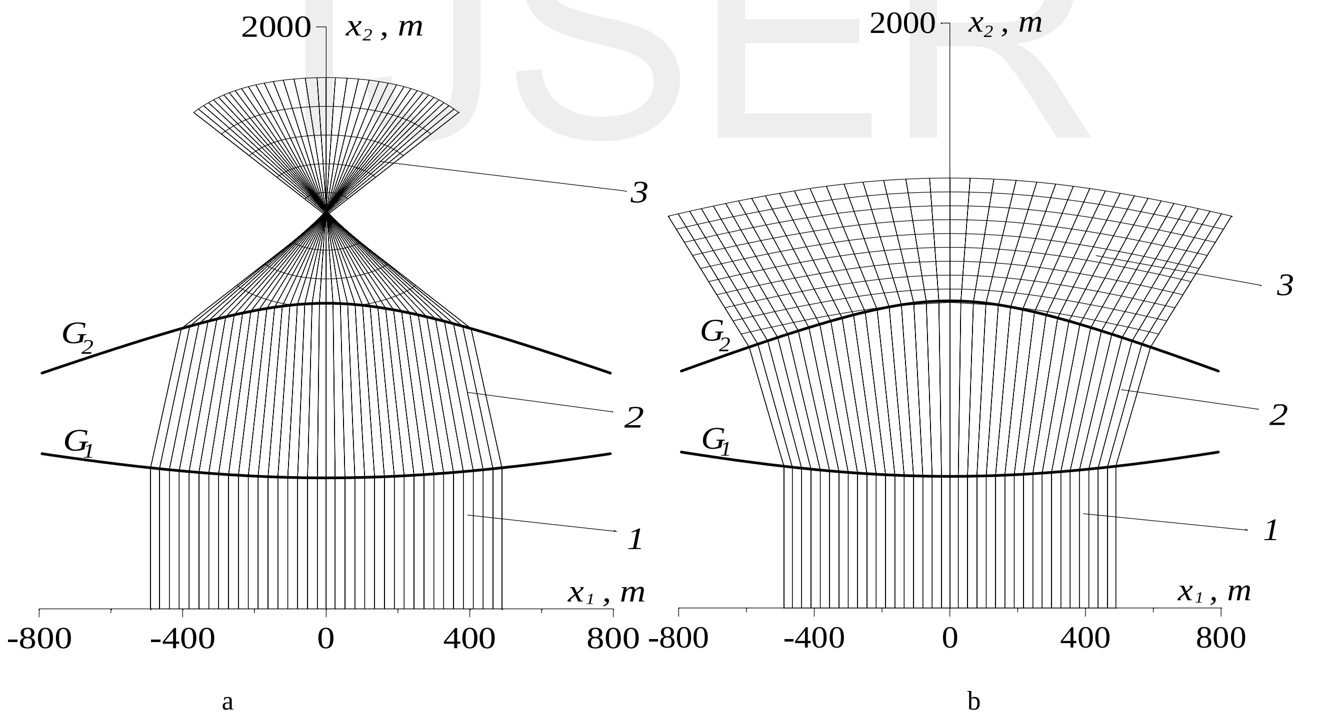
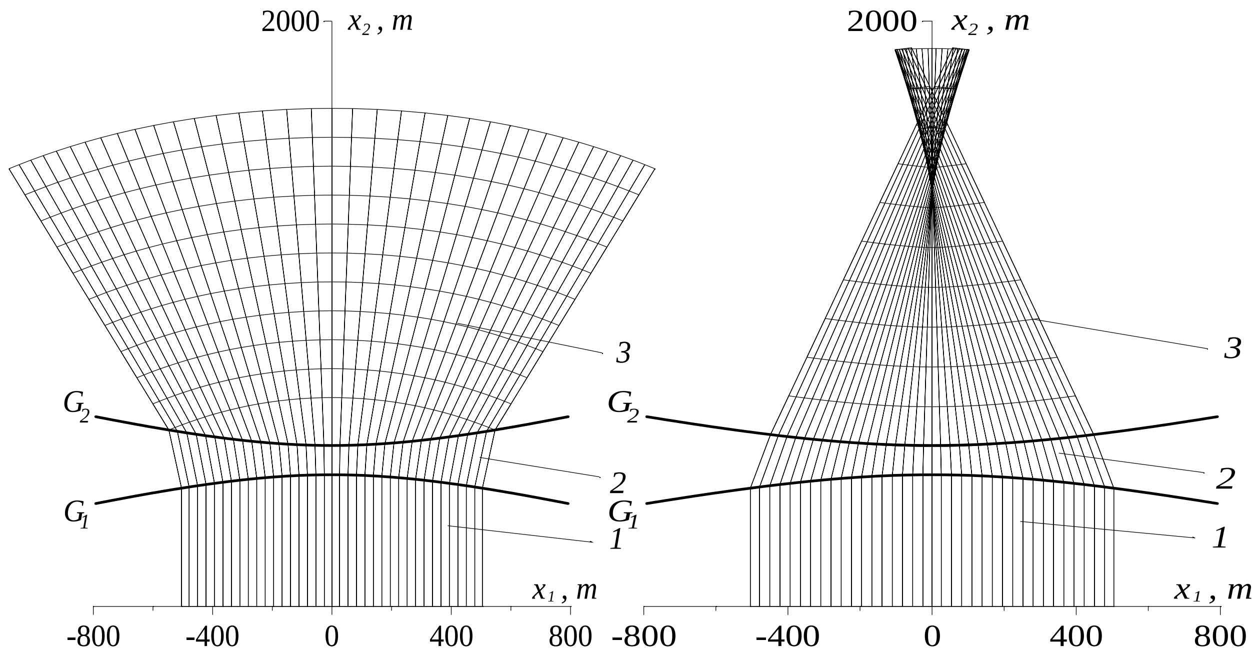
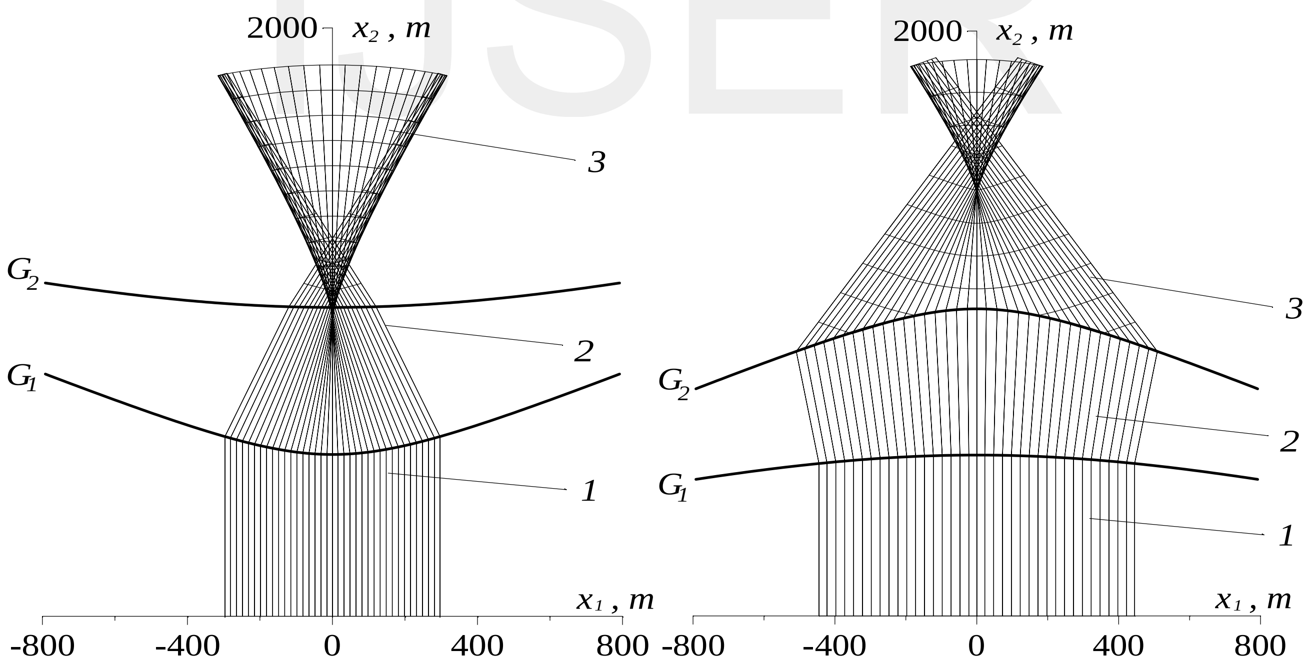


Figure 5 Focusing and scattering plane discontinuous waves by biconvex lenses.



a b

Figure 6 Scattering and focusing plane discontinuous waves by biconcave lenses.



a b

Figure 7 Focusing plane discontinuous waves by curved layers.

4. Conclusions

1. With the aid of the elaborated technique based on the ray method, the peculiarities of propagation and interaction of discontinuous waves with hyperboloid interfaces G between transversally isotropic elastic media are studied.

2. Using the ray technique, the phenomena of focusing and scattering discontinuous waves by hyperboloid free surfaces, interfaces, elastic lenses, and layers are analyzed. It is shown that abilities of the interfaces and lenses with the same geometrical parameters to focus or scatter the wave and thereby to amplify or attenuate the wave intensity are determined by the mechanical (acoustical) properties of the elastic rock media, as well as by their passage order.

3. The proposed approach to the problem of wave analysis in the vicinities of tectonic anomalies can be employed for a search for the most and least seismically dangerous zones at the Earth's surface in building constructions of high degree risk (nuclear power stations, dams, dwelling blocks, etc.).

5. References

[1] S. J. Arif, Imdadullah, M. S. J. Asghar. "Accurate measurement of velocity and acceleration of seismic vibrations near Nuclear power plants". Physics Procedia, Vol.37, 2012, 43-50.

[2] M. N. Bidgoli, L. Jing. "Anisotropy of strength and deformability of fractured rocks", Journal of Rock Mechanics and Geotechnical Engineering, Vol.6, 2014, 156-164.

[3] S. Brule, "Experiments on seismic materials: molding surface waves", Physycal review letters, Vol.112, 2014, 1-5.

[4] J. F. Dolan, B. D. Haravitch. "How well do surface slip measurements track slip at depth in large strike-slip earthquakes? The importance of fault structural maturity in controlling on-fault slip versus off fault surface deformation", Earth and Planetary Science Letters, Vol.388, 2014, 38-47.

[5] Eiby GA. Earthquakes. Heinemann: Wellington; 1980.

[6] Kasahara K. Earthquake mechanics. Cambridge University Press: Cambridge; 1981.

[7] Fedorov FI. Theory of Elastic Waves in Crystals. Plenum Press: New York; 1968.

[8] F. C. Karal, J. B. Keller. "Elastic wave propagation in homogeneous and inhomogeneous media". The Journal of the Acoustical Society of America, Vol.31, No.6, 1959, 694-705.

[9] Kravtsov YuA. Orlov Yu. I. Geometrical Optics of Heterogeneous Media. Nauka: Moscow (in Russian); 1980.

[10] J. A. Ogilvy. "A layered media model for ray propagation in anisotropic inhomogeneous materials". Applied Mathematical Modelling, Vol.14, No.5, 1990, 237-247.

[11] Arnold VI. Singularities of Caustics and Wave Fronts. Kluwer Academic publishers: The

Netherlands; 1990.

[12] Arnold VI., Varchenko AN., Gusein-Zade SM. Singularities of Differentiable Maps, Birkhauser, Boston; 1984.

[13] Poston T., Stewart I. Catastrophe Theory and its Applications. Pitman London, San Francisco: Melbourne; 1978.

[14] V. I. Gulyaev, G. M. Ivanchenko. "Focusing and dispersion of a plane wave by transversally isotropic elastic lenses", Acoustical Physics, Vol.50, No.2, 2004, 140–145.

[15] V. I. Gulyayev, G. M. Ivanchenko. "Diffraction of a plane discontinuos wave in layered anisotropic elastic media", Mechanics of Composite Materials, Vol.39, No.1, 2003, 27–36.

[16] V. I. Gulyaev, P. Z. Lugovoi, G. M. Ivanchenko. "Diffraction of discontinuous waves by ellipsoidal interfaces of transversely isotropic elastic media", International Applied Mechanics, Vol.40, No.10, 2004, 1145-1151.

[17] V. I. Gulyayev, P. Z. Lugovoi, G. M. Ivanchenko, Ye. V. Yakovenko. "The diffraction of a shock wave at the curvilinear interface of transversely isotropic elastic media". Journal of Applied Mathematics and Mechanics, Vol.64, No.3, 2000, 379-386.

[18] V. I. Gulyayev, P. Z. Lugovyy, G. M. Ivanchenko. "Discontinuous wave fronts propagation in anisotropic layered media". International Journal of Solids and Structures, Vol.40, No.1, 2002, 237-247.

[19] Petroshen, G. I. Propagation of Waves in Anisotropic Elastic Media. Nauka: Leningrad (in Russian); 1980.

[20] Podilchuk, Yu. N., Rubtsov, Yu. K. Ray Methods in Theory of Propagation and Diffraction of Waves. Naukova Dumka: Kiev (in Russian); 1988.